# BFT Hamiltonian Embedding of
# Non-Abelian Self-Dual Model

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**Abstract**

Following systematically the generalized Hamiltonian approach of Batalin, Fradkin and Tyutin, we embed the second-class non-abelian self-dual model of P. K. Townsend et al into a gauge theory. The strongly involutive Hamiltonian and constraints are obtained as an infinite power series in the auxiliary fields. By formally summing the series we obtain a simple interpretation for the first-class Hamiltonian, constraints and observables.

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# 1 Introduction

The quantization of second-class Hamiltonian systems [1] requires the strong implementation of the second-class constraints. This may imply Dirac brackets, whose non-canonical structure may pose problems on operator level. This makes it desirable to embed the second-class theory into a first-class one, where the commutator relations remain canonical and the constraints are imposed on the states. A systematic iterative procedure realizing this goal on Hamiltonian level has been developed by Batalin, Fradkin and Tyutin (BFT) [2, 3]. This procedure has been applied to a number of abelian models [4, 5, 6, 7, 8, 9, 10], where the iterative process terminates after a few steps. In the non-abelian case this iterative process may not terminate. An example is provided by the massive Yang-Mills theory [11, 12].

In this paper we consider the second-class non-abelian self-dual model of ref. [13], whose abelian version has been extensively discussed in the literature [4, 5, 7, 9, 14, 15], and we systematically construct in section 2 the first-class constraints following the BFT procedure. In section 3 we construct the observables, and in particular the Hamiltonian, as functionals of the first-class fields, and establish a simple relation between the constraints of the first- and second-class formulation. Section 4 is devoted to the interpretation of the infinite power series representing the first-class fields, by showing that the auxiliary field in power series expansion plays the role of the Lie-algebra valued fields parametrizing a non-abelian gauge transformation. In this way we establish in section 5 the connection between the Hamiltonian BFT embedding and the corresponding configuration-space embedding [16, 17].

# 2 BFT-construction of first-class constraints

Consider the self-dual Lagrangian [15],

$$\mathcal{L} = -\frac{1}{2} tr f_\mu f^\mu + \mathcal{L}_{CS} \tag{2.1}$$

where $\mathcal{L}_{CS}$ is the Chern-Simons term

$$\mathcal{L}_{CS} = \frac{1}{4m} \epsilon_{\mu\nu\rho} tr \left( f^{\mu\nu} f^\rho - \frac{2}{3} f^\mu f^\nu f^\rho \right). \tag{2.2}$$

Here $f^\mu$ are (anti-hermitian) Lie-algebra valued fields

$$f^\mu = t^a f^{\mu a} \tag{2.3}$$

and $f^{\mu\nu}$ is the usual chromoelectric field tensor

$$f^{\mu\nu} = \partial^\mu f^\nu - \partial^\nu f^\mu + [f^\mu, f^\nu]. \tag{2.4}$$

Our conventions are

$$\begin{aligned}
[t^a, t^b] &= c^{abc} t^c, \\
tr(t^a t^b) &= -\delta^{ab}.
\end{aligned} \tag{2.5}$$

The momenta canonically conjugate to $f^{0a}$ and $f^{ia}$ are respectively given by $\pi_0^a = 0$ and $\pi_i^a = -\frac{1}{2m}\epsilon_{ij}f^{ja}$. We have thus two sets of primary constraints $T_0^a = 0$, $T_i^a = 0$, with

$$\begin{aligned}
T_0^a &= \pi_0^a, \\
T_i^a &= \pi_i^a + \frac{1}{2m}\epsilon_{ij}f^{ja}.
\end{aligned} \tag{2.6}$$

The canonical Hamiltonian density associated with the Lagrangian (2.1) is given by

$$\mathcal{H}_c = -\frac{1}{2}f_\mu^a f^{\mu a} + \frac{1}{2m}f^{0a}\epsilon_{ij}f^{ija}. \tag{2.7}$$

Persistency in time of these constraints leads to one further (secondary) constraint $T_3^a = 0$, with

$$T_3^a = f^{0a} - \frac{1}{2m}\epsilon_{ij}f^{ija}. \tag{2.8}$$

The constraints (2.6) and (2.8) define a second-class system. In particular we have the Poisson-brackets

$$\left\{T_i^a, T_j^b\right\} = \frac{1}{m}\epsilon_{ij}\delta^{ab}\delta^2(x-y). \tag{2.9}$$

In order to simplify the calculations, as well as for reasons that will become apparent in section 5, we shall implement the constraints $T_i^a = 0$ strongly by introducing Dirac brackets $\{ \, , \, \}_D$ defined in the subspace of these constraints. Following the construction of Dirac [1], we have in that case, $\{T_i^a(x), T_j^b(y)\}_D = 0$, while for the remaining constraints one finds

$$\{\Omega_i^a(x), \Omega_j^b(y)\}_D = \Delta_{ij}^{ab}(x,y) \tag{2.10}$$

with

$$\Delta_{ij}^{ab}(x,y) = \begin{pmatrix} 0 & -\delta^{ab} \\ & \\ \delta^{ab} & c^{abc}\left(-\frac{1}{2m}\epsilon_{kl}f^{klc}\right) \end{pmatrix} \delta^2(x-y), \tag{2.11}$$

where we have set $T_0^a = \Omega_1^a$ and $T_3^a = \Omega_2^a$, in order to streamline the notation.

We now reduce the second-class system defined by the "commutation relations" (2.10) to a first-class system at the expense of introducing additional degrees of freedom. Following refs. [2, 3], we introduce auxiliary fields $\Phi^{1a}$ and $\Phi^{2b}$ corresponding to $\Omega_1^a$ and $\Omega_2^a$, with the Poisson bracket

$$\left\{\Phi^{ia}(x), \Phi^{jb}(y)\right\}_D = \omega_{ab}^{ij}\delta^2(x-y), \tag{2.12}$$

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where we are free [2] to make the choice

$$\omega_{ab}^{ij} = \epsilon^{ij}\delta_{ab}. \tag{2.13}$$

The first-class constraints $\tilde{\Omega}_i^a$ are now constructed as a power series in the auxiliary fields,

$$\tilde{\Omega}_i^a = \Omega_i^a + \sum_{n=1}^{\infty} \Omega_i^{(n)a} \tag{2.14}$$

where $\Omega_i^{(n)a}(n=1,...,\infty)$ are homogeneous polynomials in the auxiliary fields $\{\Phi^{jb}\}$ of degree $n$, to be determined by the requirement that the constraints $\tilde{\Omega}_i^a$ be strongly involutive:

$$\left\{\tilde{\Omega}_i^a(x),\tilde{\Omega}_j^b(y)\right\}_D = 0. \tag{2.15}$$

Making the Ansatz

$$\Omega_i^{(1)a}(x) = \int d^2y X_{ij}^{ab}(x,y)\Phi^{jb}(y) \tag{2.16}$$

and substituting (2.14) into (2.15) leads to the condition

$$\int d^2z d^2z' X_{ik}^{ac}(x,z)\omega_{cd}^{kl}(z,z')X_{j\ell}^{bd}(z',y) = -\Delta_{ij}^{ab}(x,y). \tag{2.17}$$

For the choice (2.13) for $\omega_{ab}^{ij}$, equation (2.17) has (up to a natural arbitrariness) the solution

$$X_{ij}^{ab}(x,y) = \begin{pmatrix} \delta^{ab} & 0 \\ & \\ \frac{1}{4m}c^{abc}\epsilon_{kl}f^{ijc} & \delta^{ab} \end{pmatrix} \delta^2(x-y). \tag{2.18}$$

Substituting (2.18) into (2.16) as well as (2.14), and iterating this procedure one finds the strongly involutive first-class constraints to be given by

$$\tilde{\Omega}_1^a = \pi_0^a + \Phi^{1a} \tag{2.19}$$

$$\tilde{\Omega}_2^a = f^{0a} - \frac{1}{2m}\epsilon_{ij}f^{ija} + \Phi^{2a}$$
$$- \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)!}\left[\left(\tilde{\Phi}^1\right)^n\right]^{ab}\left(\frac{1}{2m}\epsilon_{ij}f^{ijb}\right), \tag{2.20}$$

where

$$\left(\tilde{\Phi}^1\right)^{ab} = c^{acb}\Phi^{1c}. \tag{2.21}$$

It turns out convenient to define the field

$$V(\theta) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)!}\theta^n \tag{2.22}$$

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where $\tilde{\theta}$ is a Lie-algebra valued field in the adjoint representation, $\tilde{\theta} = \theta^a T^a$, with $T^c_{ab} = c^{acb}$. In terms of $V(\theta)$ the constraint $\tilde{\Omega}^a_2$ reads

$$\tilde{\Omega}^a_2 = f^{0a} + \Phi^{2a} - V^{ab}(\Phi^1)\left(\frac{1}{2m}\epsilon_{ij}f^{ijb}\right). \tag{2.23}$$

This completes the construction of the first-class constraints.

# 3 Construction of first-class fields

The construction of the first-class Hamiltonian $\tilde{H}$ can be done along similar lines as in the case of the constraints, by representing it as a power series in the auxiliary fields and requiring $\{\tilde{\Omega}^a_i, \tilde{H}\}_D = 0$ subject to the condition $\tilde{H}[f, \Phi = 0] = H_c$. We shall follow here a somewhat different path [6] by noting that any functional of first-class fields $\tilde{f}^\mu$ will also be first-class. This leads us to the identification $\tilde{H} = H_c[\tilde{f}]$. The "physical" fields $\tilde{f}^\mu$ are obtained as a power series in the auxiliary fields $\Phi^{ia}$ by requiring them to be strongly involutive: $\{\tilde{\Omega}^a_i, \tilde{f}^\mu\}_D = 0$. The iterative solution of these equations involves the use of (2.13) and (2.18) and leads to an infinite series which can be compactly written in terms of $\tilde{\Phi}^1$ defined in (2.21) as

$$\begin{aligned}
\tilde{f}^{0a} &= f^{0a} + \Phi^{2a} + \left(U^{ab}(\Phi^1) - V^{ab}(\Phi^1)\right)\left(\frac{1}{2m}\epsilon_{ij}f^{ijb}\right), \\
\tilde{f}^{ia} &= U^{ab}(\Phi^1)f^{ib} + V^{ab}(\Phi^1)\partial^i\Phi^{1b},
\end{aligned} \tag{3.1}$$

where $V(\theta)$ has been defined in (2.22) and $U(\theta)$ is given by

$$U(\theta) = 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{n!}\tilde{\theta}^n. \tag{3.2}$$

For $\tilde{\pi}^a_0$ we have correspondingly

$$\tilde{\pi}^a_0 = \pi^a_0 + \Phi^{1a}. \tag{3.3}$$

We now observe that the first-class constraints (2.19, 2.20) can be written in terms of the physical fields as

$$\begin{aligned}
\tilde{\Omega}^a_1 &= \tilde{\pi}^a_0 \\
\tilde{\Omega}^a_2 &= \tilde{f}^{0a} - \frac{1}{2m}\tilde{f}^{0a}\epsilon_{ij}\tilde{f}^{ija}.
\end{aligned} \tag{3.4}$$

Comparing with the second-class constraints $T^a_0$ and $T^a_3$ in eqs. (2.6) and (2.8), we see that the first-class constraints (3.4) are just the second-class constraints written in terms of the physical variables. Correspondingly, we take the first-class Hamiltonian density $\tilde{\mathcal{H}}$ to be given by the second-class one (2.7), expressed in terms of the physical fields:

$$\tilde{\mathcal{H}} = -\frac{1}{2}\tilde{f}^a_\mu\tilde{f}^{\mu a} + \frac{1}{2m}\tilde{f}^{0a}\epsilon_{ij}\tilde{f}^{ija}. \tag{3.5}$$

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It is important to notice that any Hamiltonian weakly equivalent to (3.5) describes the same physics since the observables of the first-class formulation must be first-class themselves. Hence we are free to add to $\tilde{\mathcal{H}}$ any terms proportional to the first-class constraints.

## 4    Interpretation of the results

For what follows it will be convenient to rewrite the constraints (2.19) and (2.20), as well as $\tilde{f}^{\mu a}$ in (3.1) in terms of canonically conjugate fields. To this end we observe that the symplectic structure (2.12) allows for the identifications $\Phi^{1a} = \theta^a$, $\Phi^{2a} = \pi_\theta^a$, with $(\theta^a, \pi_\theta^a)$ canonically conjugate pairs. In this notation the constraints $\tilde{\Omega}_i^a \approx 0$ and the fields $\tilde{f}^{\mu a}$ take, respectively, the form

$$\tilde{\Omega}_1^a = \pi_0^a + \theta^a,$$
$$\tilde{\Omega}_2^a = f^{0a} + \pi_\theta^a - V^{ab}(\theta)(\frac{1}{2m}\epsilon_{ij}f^{ijb}), \tag{4.1}$$

and

$$\tilde{f}^{0a} = f^{0a} + \pi_\theta^a + (U^{ab}(\theta) - V^{ab}(\theta))(\frac{1}{2m}\epsilon_{ij}f^{ijb}),$$
$$\tilde{f}^{ia} = U^{ab}(\theta)f^{ib} + V^{ab}(\theta)\partial^i\theta^b. \tag{4.2}$$

For the first-class field strength tensor $\tilde{f}^{ija}$ one has correspondingly

$$\tilde{f}^{ija} = U^{ab}(\theta)f^{ijb}. \tag{4.3}$$

The field $\tilde{f}^{ia}$ has a simple interpretation. Defining the group valued field

$$g(\theta) = e^\theta, \quad \theta = \theta^a t^a \tag{4.4}$$

we have for a Lie algebra valued field $A = A^a t^a$,

$$- tr(t^a g^{-1}(\theta)Ag(\theta)) = U^{ab}(\theta)A^b$$
$$-tr(t^a g^{-1}(\theta)\partial_\mu g(\theta)) = V^{ab}(\theta)\partial_\mu\theta^b. \tag{4.5}$$

The l.h.s. of these equations resumes in compact form the infinite series on the r.h.s.. We thus see that the expression for $\tilde{f}^i = \tilde{f}^{ia}t^a$ in (4.2) can correspondingly be written in the compact form

$$\tilde{f}^i = g^{-1}f^i g + g^{-1}\partial^i g. \tag{4.6}$$

The fields $\tilde{f}^{ia}$ are thus identified with the gauge-transform of the fields $f^{ia}$. They are invariant under the extended gauge transformation

$$f^i \to h^{-1}f^i h + h^{-1}\partial^i h$$
$$g \to h^{-1}g \tag{4.7}$$

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and thus represent natural observables in the extended space. Correspondingly, the first-class field strength tensor $\tilde{f}^{ij} = \tilde{f}^{ija}t^a$ takes the form

$$\tilde{f}^{ij} = g^{-1}f^{ij}g. \tag{4.8}$$

Since the field strength tensor transforms homogeneously under gauge transformations, this is in agreement with our expectations and suggests that we should have the weak equality

$$\tilde{f}^0 \approx g^{-1}f^0 g + g^{-1}\partial^0 g. \tag{4.9}$$

We now demonstrate this. To this end we observe that $\tilde{f}^{0a}$ in (4.2) can be written as

$$\tilde{f}^{0a} = U^{ab}(\theta)(\frac{1}{2m}\epsilon_{ij}f^{ijb}) + \tilde{\Omega}_2^a. \tag{4.10}$$

Hence recalling (4.5), we have

$$\tilde{f}^0 \approx g^{-1}(\frac{1}{2m}\epsilon_{ij}f^{ij})g. \tag{4.11}$$

From (4.8) we see that this is nothing but the second-class constraint (2.8) with $f^\mu$ replaced by $\tilde{f}^\mu$.

The above considerations indicate that $\tilde{f}^\mu$ is weakly equal to nothing but the gauge transform of $f^\mu$, with $g$ in the gauge group. In order to put this claim on a solid basis, we consider the gauge-transform of the Lagrangian (2.1) as given by

$$\hat{\mathcal{L}} = -\frac{1}{2}tr(\hat{f}^\mu \hat{f}_\mu) + \frac{1}{4m}\epsilon_{\mu\nu\rho}tr(f^{\mu\nu}f^\rho - \frac{2}{3}f^\mu f^\nu f^\rho) \tag{4.12}$$

with $\hat{f}^\mu = g^{-1}f^\mu g + g^{-1}\partial^\mu g$, where we have made use of the fact that the gauge transformation leaves the Chern-Simons action invariant, up to a topological term and a surface term:

$$\hat{\mathcal{L}}_{CS}(f, g) = \mathcal{L}_{CS}(f) - \frac{2}{3}tr(g^{-1}dg)^3 + 2\epsilon_{\mu\nu\rho}\partial^\rho(f^\mu \partial^\nu gg^{-1}). \tag{4.13}$$

Making use of (4.5) we may write $\hat{f}^\mu$ in the form

$$\hat{f}^{\mu a} = U^{ab}(\theta)f^{\mu b} + V^{ab}(\theta)\partial^\mu \theta^b. \tag{4.14}$$

For the momenta canonically conjugate to $f^{\mu a}$ and $\theta^a$ one finds

$$\pi_0^a = 0, \quad \pi_i^a = -\frac{1}{2m}\epsilon_{ij}f^{ja}$$
$$\pi_\theta^a = [U^{bc}(\theta)f^{0c} + V^{bc}(\theta)\partial^0\theta^c]V^{ba}(\theta). \tag{4.15}$$

The first two relations represent primary constraints. Note that the canonical momenta and fields are not to be confused with those of the second-class formulation. The Hamiltonian corresponding to (4.12) takes the form

$$\begin{aligned}\hat{\mathcal{H}} &= \frac{1}{2m}f^{0a}\epsilon_{ij}f^{ija} + \frac{1}{2}(U^{ab}(\theta)f^{ib} + V^{ab}(\theta)\partial^i\theta^b)^2 \\ &\quad + \frac{1}{2}(\pi_\theta^b(V^{-1}(\theta))^{ba})^2 - \pi_\theta^b(V^{-1}(\theta))^{ba}U^{ac}f^{0c}.\end{aligned} \tag{4.16}$$

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Persistency in time of the primary constraint $\hat{\Omega}_1^a = \pi_0^a = 0$ leads to the secondary constraint

$$\hat{\Omega}_2^a = \pi_\theta^b (V^{-1}(\theta))^{bc} U^{ca}(\theta) - \frac{1}{2m} \epsilon_{ij} f^{ija}, \tag{4.17}$$

while a similar requirement for the constraints $\pi_i^a + \frac{1}{2m} \epsilon_{ij} f^{ja} = 0$ generates no further constraints, but merely serves to fix the corresponding Lagrange multipliers in the total Hamiltonian. As before we implement these constraints strongly. With respect to the corresponding Dirac brackets, the other constraints $\hat{\Omega}_i^a \approx 0, \quad i = 1, 2$ are first-class, and thus reflect the underlying gauge invariance of the Lagrangian (4.12). We now return to (4.14). From (4.14) it immediately follows that

$$\hat{f}^{ia} = \tilde{f}^{ia} \tag{4.18}$$

Making use of (4.15) in order to eliminate $\partial^0 \theta^a$ in favor of $\pi_\theta^a$, we further have

$$\hat{f}^{0a} = \pi_\theta^b (V^{-1}(\theta))^{ba} = U^{ab}(\theta)(\frac{1}{2m} \epsilon_{ij} f^{ijb}) + U^{ab}(\theta)\hat{\Omega}_2^b \tag{4.19}$$

or equivalently

$$\hat{f}^0 = g^{-1}(\frac{1}{2m} \epsilon_{ij} f^{ij} + \hat{\Omega}_2)g. \tag{4.20}$$

Hence, comparing with (4.11), we conclude

$$\hat{f}^0 \approx \tilde{f}^0 \tag{4.21}$$

This establishes the weak equivalence of $\hat{f}^\mu$ and $\tilde{f}^\mu$. We furthermore have from (4.17)

$$V^{ab}(\theta)\hat{\Omega}_2^b = \pi_\theta^a - V^{ab}(\theta)(\frac{1}{2m} \epsilon_{ij} f^{ijb}), \tag{4.22}$$

where we have made use of the identity

$$U^{ac}(\theta)V^{bc}(\theta) = V^{ab}(\theta). \tag{4.23}$$

Performing the canonical transformation

$$\pi_\theta^a \rightarrow \pi_\theta^a + f^{0a}$$
$$\pi_0^a \rightarrow \pi_0^a + \theta^a \tag{4.24}$$

the first-class constraints $\hat{\Omega}_0^a \approx 0$ and $V^{ab}\hat{\Omega}_3^b \approx 0$ map into the constraints (4.1). It remains to check the relation between $\hat{\mathcal{H}}$ and $\tilde{\mathcal{H}}$. Making use of (4.19), expression (4.16) for $\hat{\mathcal{H}}$ may be written in the form

$$\hat{\mathcal{H}} = \frac{1}{2}(U^{ab}(\theta)f^{ib} + V^{ab}(\theta)\partial^i\theta^b)^2 + \frac{1}{2}(\frac{1}{2m} \epsilon_{ij} f^{ija})^2$$
$$+ \frac{1}{2}(\hat{\Omega}_3^a)^2 + (\frac{1}{2m} \epsilon_{ij} f^{ija} - f^{oa})\hat{\Omega}_3^a. \tag{4.25}$$

Comparison of (4.25) with (3.5) immediately shows that $\hat{\mathcal{H}} \approx \tilde{\mathcal{H}}$. We thus conclude that the BFT constructioon is equivalent to the quantization of the gauge theory defined by the Lagrangian (4.12).

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# 5 Revisiting section 4

The discussion of section 4 indicates that there exists a more economical path for arriving at the results. It consists in gauging the Lagrangian (2.1) by making the substitution $f^\mu \to \hat{f}^\mu$. The resulting Lagrangian, eq. (4.12), can be written in the form

$$\hat{\mathcal{L}}(f,g) = \mathcal{L}(f) + \mathcal{L}_{WZ} \tag{5.1}$$

where

$$\mathcal{L}_{WZ}(f,g) = -tr(f^\mu \partial_\mu g g^{-1}) - \frac{1}{2} tr(g^{-1}\partial^\mu g)^2 \tag{5.2}$$

plays the role of the Wess-Zumino-Witten (WZW) term in the gauge-invariant formulation of two-dimensional chiral gauge theories [16, 17, 18], and $\mathcal{L}(f)$ is the Lagrangian of the second-class system. Contrary to what was done in section 4, we choose to work here with the group valued field $g$, instead of the Lie-algebra valued field $\theta$. We then have for the momentum $\Pi$ conjugate to $g$,

$$\Pi^T = -g^{-1}f^0 - g^{-1}\partial^0 g g^{-1} \tag{5.3}$$

where "T" denotes "transpose". The canonical momenta $\pi_\mu$ are the same as before. Hence the primary constraints are still of the form (2.6),

$$\hat{T}_0^a = \pi_0^a, \quad \hat{T}_i^a = \pi_i^a + \frac{1}{2m}\epsilon_{ij}f^{ija} \tag{5.4}$$

though the dynamics is a different one. The canonical Hamiltonian corresponding to (5.1) reads, on the constraint surface $\pi_0^a = 0$,

$$
\begin{aligned}
H_C = \int d^2x \Big\{ &-\frac{1}{2}tr(\Pi^T g)^2 - \frac{1}{2}tr f_i^2 + tr(f^i g \partial_i g^{-1}) \\
&-\frac{1}{2}tr(g^{-1}\partial^i g)^2 + tr f^0(\partial^i \hat{T}_i + \hat{T}_3) \Big\}
\end{aligned} \tag{5.5}
$$

where $\hat{T}_3$ is given by

$$\hat{T}_3 = -\frac{1}{2m}\epsilon_{ij}f^{ij} - g\Pi^T. \tag{5.6}$$

The requirement $\dot{\pi}_0^a = 0$ leads to the secondary constraint $\hat{T}_3^a + \partial^i \hat{T}_i = 0$.

The constraints $\hat{T}_i^a = 0$ are evidently second class, and as before we implement them strongly by working with the corresponding Dirac brackets. On the surface defined by $\hat{T}_i = 0$, the secondary constraints read $\hat{T}_3^a = 0$. One easily checks that the constraints $\hat{T}_0^a = 0$ and $\hat{T}_3^a = 0$ are first class with respect to these Dirac brackets.

It remains to establish the relation with the results of section 4. Multiplying (5.3) from the right with $g$ and using (4.5) we have

$$
\begin{aligned}
tr(t^a g \Pi^T) &= f^{0a} + V^{ab}(-\theta)\partial^0 \theta^b \\
&= f^{0a} + U^{ca}(\theta)V^{cb}(\theta)\partial^0 \theta^b, \tag{5.7}
\end{aligned}
$$

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where we have further used

$$U^{ca}(\theta)V^{cb}(\theta) = V^{ab}(-\theta). \tag{5.8}$$

Comparing (5.7) with (4.15) and making further use of

$$U^{ba}(\theta)U^{ca}(\theta) = \delta^{ab} \tag{5.9}$$

we see that

$$V^{ac}(\theta)U^{ab}(\theta)tr(t^b g\Pi^T) = \pi_\theta^c \tag{5.10}$$

On the other hand, from (5.6) we deduce

$$U^{ab}(\theta)tr(t^b g\Pi^T) = -U^{ab}(\theta)(\frac{1}{2m}\epsilon_{ij}f^{ija} + \hat{T}_3^a). \tag{5.11}$$

Let us compare this with $\hat{f}^0$ defined by

$$\hat{f}^0 = g^{-1}f^0 g + g^{-1}\partial^0 g. \tag{5.12}$$

From (5.3) we see that

$$\hat{f}^0 = -\Pi^T g. \tag{5.13}$$

Using (5.6) and noting that $\hat{T}_0 = \hat{\Omega}_1, \hat{T}_3 = \hat{\Omega}_2$, we recover (4.20). This establishes the equivalence of the various procedures.

# 6    Conclusion

The main objective of this paper was to provide a nontrivial example for the Hamiltonian embedding of a second-class theory into a first-class one, following the systematic constructive procedure of Batalin, Fradkin, and Tyutin ([2, 3]). Unlike the case of the abelian models discussed in the literature, the first-class Hamiltonian and secondary constraint generated by this procedure are obtained as an infinite power series in the auxiliary fields living in the extended phase space. By explicitly summing this series we established the weak equivalence with the corresponding quantities as obtained by gauging the second-class Lagrangian defining our model, with the auxiliary fields playing the role of the corresponding gauge degrees of freedom. We thereby showed that on the space of gauge-invariant functionals the Lagrangian approach of refs. [16, 17] for embedding second-class theories into a gauge theory is equivalent to the Hamiltonian BFT approach. We further showed that the most economical way of obtaining the desired results would consist in working with the group rather than Lie-algebra valued fields of the gauged Lagrangian. One readily checks that the same conclusion can be drawn for the model of ref. [11].

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